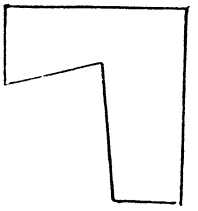
ROUND I: Definitions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. A *verjoin* of a polygon is a line segment joining two vertices not joined by a single side. How many verjoins does the figure shown have?



2. The function S is a 5-card shuffle function defined by S(1,2,3,4,5) = (3,1,4,2,5). What is the smallest number of consecutive applications of S that will return any 5-card ordered set to its original order?

3. A figure is informally described as convex if it has no dents. Formally we define a figure as *convex* iff for every pair of points in its interior, the connecting segment is also within its interior. For example, the figure at the right is not convex. Which of the following statements would also work to define *convex polygons*? A polygon is *convex* iff...

A) all diagonals lie in the interior, except for their endpoints
B) all diagonals are longer than all of the sides
C) the figure's perimeter is also the shortest path which encloses the entire figure
D) none of the lines containing the sides passes through the interior
E) no angle has to be measured outside to get a measure less than 180 deg.
F) the largest interior angle is adjacent to the longest side

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Mass. Academy, Worcester Academy, QSC

December 2, 1998 WOCOMAL VARSITY MEET

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve for y: $x^2 + y - 16 = 0$ and $y = x^2 + 4$

2. If 34 times half of a number exceeds the square of the number by 72, what are all possible values for the number?

3. Angela jogged at 6 miles/hour for one third of the distance of her morning jog and at 8 miles/hour for two thirds of the distance. What was her average speed for the whole trip?

ANSWERS

(1 pt) 1. y = _____

(2 pts) 2. _____

(3 pts) 3. _____ miles/hr

Doherty, Tantasqua, Worcester Academy

ROUND III: Problem Solving

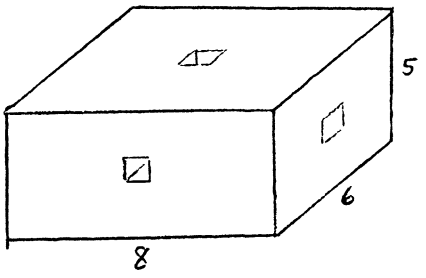
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Consider a number such that you get the same answer by subtracting 5 from it as you do by multiplying it by 5. Which of the following are true aboout such a number?
   A) There is no such number.
   B) There is exactly one such number.
   C) There are two such numbers.
   D) There are infinitely many such numbers.
   E) It (they) is (are) negative.

2 and 3. An 8 by 6 by 5 rectangular solid has three holes with 1 by 1 square cross sections drilled through it connecting the centers of opposite faces and crossing inside in a cube shape. For the resulting object,

2. Find the volume.

3. Find the surface area. (Include the inside surfaces.)



ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____ units^3

(3 pts) 3. _____ units^2

Tantasqua, Worcester Academy, QSC

ROUND IV: Sequences and series

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Evaluate $\sum_{n=1}^{10}(-1)^n(2n-1)$

2. What number may be added to each of 1, 4, and 2, in that order, so that the resulting numbers will form a geometric progression?

3. An arithmetic sequence has 12 terms. The sum of the first 8 terms is equal to the sum of the last 4 terms. If the first term is 15, find the last term.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Bancroft, Hudson, St.John's

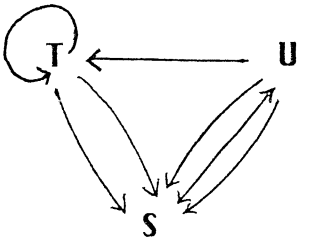
ROUND U: Matrices

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If the dimessions of matrices A, C, D, and E are as given, what must be the dimensions of matrix B for the noted multiplication to be possible?

   A: 5 by 3 D: 2 by 4
   C: 3 by 4 E: 5 by 2 ACBDB = E

2. Fill in the adjacency matrix for the directed graph shown.



3. In a three team track meet, Marlboro had 4 first place finishes, 9 second place finishes, and 4 third place finishes. Hudson had 7 firsts, 5 seconds, and 7 thirds. Shrewsbury had 8 firsts, 5 seconds, and 8 thirds. 5 points are awarded for first, 3 points for second, and 1 point for third. Express all this numerical information in the form of a product of two matrices and work out that product so that the result shows the total point scores for Marlboro, Hudson, and Shrewsbury in that order from top to bottom.

ANSWERS

(1 pt) 1. _____ by _____

FROM \ TO →	S	T	U
S			
T			
U			

(2 pts) 2.

$$\begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

(3 pts) 3.

Doherty, Quaboag, Shrewsbury

December 2, 1998 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open

2 points each

ALL ANSWERS MUST BE IN SIMPLEST FORM

1. Let a number be called eventending if the number of its even divisors exceeds the number of its odd divisors. Among the first 25 positive integers, how many are eventending?

2. Find the smallest real value of x for which $\sqrt{4x^2+12x+9} = 2x + 3$.

3. Suppose f(x,y) = (x+y, x-y) is a function from 2 variables to 2 variables. For points A(a,m) and C(f(f(a,m))), find distance AC in terms of a and m.

4. For failing to complete a bridge in a specified time, a contractor is compelled to forfeit $100 a day for each day of the first ten days of extra time required; and for each additional day, beginning with the eleventh, the forfeit is increased by $10 a day. He loses a total of $2980. How many days did he overrun the stipulated time?

5. Given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ k & 3 & 5 \\ 4 & 5 & 12 \end{bmatrix}$ and determinant of A = 3. In the inverse matrix for A, if $b_{13} = 1/3$ and $b_{33} = 1/3$, what is the value of b_{23}? b_{ij} is the entry in row i and column j of A^{-1}.

6. A rectangular sheet of paper is red on the top side and green on the bottom side. The lower right-hand corner is folded to the left side of the paper so that the resulting figure is a red rectangle above a green right triangle. The width of the peper is 8 inches and equal areas of red and green appear after the folding. What is the area of one side of the paper?

7. A line segment joins P(3,6) and Q(-1,4). The segment is first reflected in the line y = x. That image is then rotated 90° counterclockwise about the origin and the result is then reflected in the y-axis. What is the slope of the final image?

8. When the product of two numbers is added to the sum of their reciprocals, the answer is 5.9. If the sum of the two numbers is 2.7, what is the largest possible value of the product of the two numbers?

9. Find n if $\left(10^{12}+25\right)^2 - \left(10^{12}-25\right)^2 = 10^n$.

Assabet Valley, Auburn,Bromfield, Doherty, St.John's, Worcester Academy

December 2, 1998 — WOCOMAL Varsity Meet ANSWERS

Round I — defs

1. (1 pt) 9
2. (2 pts) 4
3. (3 pts) A, C, D, E — need all 4

Round II — alg 1

1. (1 pt) 10
2. (2 pts) 8 and 9
3. (3 pts) 7.2 or $7\frac{1}{5}$

Round III — prob solv

1. (1 pt) B, E — need both
2. (2 pts) 223
3. (3 pts) 294

Round IV — seq series

1. (1 pt) 10
2. (2 pts) -2.8 or $-2\frac{4}{5}$ or $-\frac{14}{5}$
3. (3 pts) 81

Round V — matrix

1. (1 pt) 4 by 2
2. (2 pts) $\begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 0 \end{bmatrix}$
3. (3 pts) $\begin{bmatrix} 4 & 9 & 4 \\ 7 & 5 & 7 \\ 8 & 5 & 8 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 51 \\ 57 \\ 63 \end{bmatrix}$ — Need 3 matrices

TEAM ROUND 2 pts each

1. 6
2. -1.5 or $-\frac{3}{2}$ or $-1\frac{1}{2}$
3. $\sqrt{a^2+m^2}$
4. 22
5. $-\frac{2}{3}$
6. 96
7. $\frac{1}{2}$ or .5
8. 5.4 or $5\frac{2}{5}$ or $\frac{27}{5}$
9. 14